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B.A./B.Sc. SIXTH SEMESTER EXAMINATION, JUNE 2022
THIRD YEAR [BATCH 2019-22]
PHYSICS (HONOURS)
Paper : XIII [CC13]

Date : 15/06/2022
Time : 11 am – 1 pm
Full Marks : 50

Answer **any five** questions: [5×10]

1. a) Distinguish between microcanonical ensemble and canonical ensemble. What is the postulate of "equal a priori probability". Is it valid for canonical system? Justify your answer. [2+2]

   b) For a microcanonical ensemble find out the expression of temperature (T), pressure (P) and chemical potential (μ) in terms of number of accessible states (Ω). For an ideal non-relativistic gas of N molecules confined in a volume V, $\Omega \propto \left(E^{\frac{3N}{2}}.V^N\right)$. Show that for reversible adiabatic process, $PV^{\frac{5}{3}}$ = constant. [3+3]

2. a) State the classical equipartition theorem of energy. Comment on its limit of validity. Estimate the rotational kinetic energy of a diatomic molecule in ev, when it is in thermal equilibrium at a temperature 300 K. [1+1+2]

   b) Because of the Doppler effect, the frequency of the light observed from an atom having an x-component of velocity (V_x) is $v \cong v_0\left(1+\frac{v_x}{c}\right)$, v_0 and c are constants.
   Calculate the dispersion $<(\Delta v)^2>=<(v-<v>)^2>$ [3]

   c) A canonical system has two non-degenerate energy levels $E_o = 0$ and $E_1 = 0.5ev$. Estimate the temperature at which 1% of the total population occupy the higher level. [Boltzmann constant $k_B = 8.625\times10^{-5}ev.K^{-1}$] [3]

3. a) Write down the expression of grand canonical partition function Z. Using Z find the expression of the mean particle number and mean energy of the system. [1+3]

   b) Find the expression of entropy of a grand canonical system using Shanon's formula. Utilise that value of entropy, to find the value of $(-k_B T \ln Z)$. Now define Grand Potential (Φ_G). Show that $d\Phi_G = -pdV - SdT - Nd\mu$ [2+2+2]

4. a) An one-dimensional Quantum Harmonic Oscillator is in equilibrium with a heat reservoir at temperature T. Derive its partition function. Then find the average energy of the oscillator. Hence calculate the specific heat of the oscillator and show its variation with temperature (T) in a graph. [3+1+1+1]

   b) A system of N ideal gas molecule is in contact with a heat bath at temperature T. The single particle partition function of the gas molecule is $Z_1 = CVT^{\frac{5}{2}}$, where C is a constant. Calculate the average energy and heat capacity at constant volume of the system. [4]

5. a) What are the characteristic feature of (i) bosons and (ii) fermions? Draw the nature of their distribution functions. [2+2]

b) Explain why chemical potential of boson is always negative. Discuss the approximate variation of chemical potential of bosons at extremely low temperature. [2+2]

c) What do you know about the cosmic microwave background radiation? [2]

6. a) A photon gas is enclosed in a volume V and in thermal equilibrium at temperature T.

i) Calculate the density of states of the radiation field as function of frequency (v). Hence derive Plank's law [energy per unit volume u(v) of the radiation field] and plot its nature. [2+2+1]

ii) Staring from Planck's law establish Stefan's law. Utilise Stefan's law to find the specific heat at constant volume and entropy of the radiation field. [2+3]

7. a) Consider a non-interacting Fermi gas at temperature T = 0 K. Derive the expression of Fermi energy (E_F), average energy per fermion and the degeneracy pressure of the Fermi gas. [5]

b) Find the specific heat of the free electrons in metals. [5]

8. a) A drunk person starts from the origin and moves randomly along x or -x direction. Each step is of equal length *l*. the probability that any one step along x or along -x are same and equal to half ($\frac{1}{2}$). Suppose that the mean time taken by the person for each step is $\tau^* = t / N$, where *t* is the time taken for N steps.

(i) Estimate the probability $P_N(r)$ that the displacement (x) of the person from the origin is equal to r*l* (r is any positive integer) out of N steps. [2]

(ii) Find out the value of $\langle r^2 \rangle$ and show that $x_{r.m.s}$ is proportional to $t^{\frac{1}{2}}$. [2+1]

b) Show that for a two-dimensional(no-relativistic) free electron gas at temperature T, the number of free electrons per unit area is given by

$$n = \frac{4\pi n k_B T}{h^2} \, l\,\mathrm{n}\left[1 + \exp\left(\frac{E_F}{k_B T}\right)\right]$$

[symbols are of usual significances] [5]

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